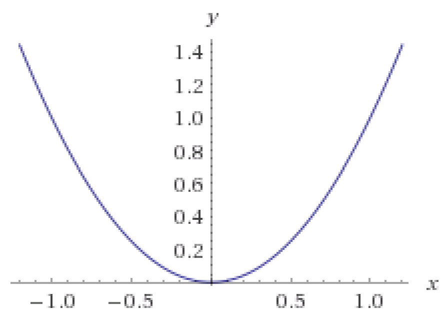
Review: Properties of Graphs - 9/14/16

1 Increasing/Decreasing

Definition 1.0.1 *A function f is **increasing** on an interval I if $f(b) > f(a)$ whenever $b > a$ for all points a and b in I that are in the domain of f.*

Definition 1.0.2 *A function f is **decreasing** on an interval I if $f(b) < f(a)$ whenever $b > a$ for all points a and b in I that are in the domain of f.*

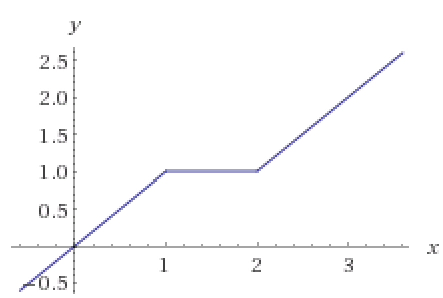
Example 1.0.3 *The following graph is increasing on $(0, \infty)$. It is decreasing on $(-\infty, 0)$.*



Definition 1.0.4 *A function f is **weakly increasing** on an interval I if $f(b) \geq f(a)$ whenever $b > a$ for all points a and b in I that are in the domain of f.*

Definition 1.0.5 *A function f is **weakly decreasing** on an interval I if $f(b) \leq f(a)$ whenever $b > a$ for all points a and b in I that are in the domain of f.*

Example 1.0.6 *The following function is weakly increasing:*



2 Intercepts

Definition 2.0.7 *A function has a* ***y intercept*** *at the point where it crosses the y axis. A point on a function is a y-intercept if and only if the x coordinate is 0.*

Definition 2.0.8 *A function has an* ***x intercept*** *at the point where it crosses the x axis. A point on a function is an x-intercept if and only if the y coordinate is 0.*

Example 2.0.9 *Let $f(x) = x^2 + 2$. What are the x and y intercepts?*
To find the y intercept, plug 0 in for x:
$y = 0^2 + 2 = 2$, so y intercept is at $(0, 2)$.

To find the x intercept, plug 0 in for y:
$0 = x^2 + 2$, so $x^2 = -2$. This is impossible, so we don't have an x intercept.

Example 2.0.10 *Let $f(x) = x^2 - 5x + 4$. What's the y intercept? We plug 0 in for x to get that $y = 4$, so the intercept is at $(0, 4)$. What about the x intercept? We plug 0 in for y to get $0 = x^2 - 5x + 4 = (x - 1)(x - 4)$, so we have two x intercepts, $(1, 0)$ and $(4, 0)$.*

Practice Problems
Find the x and y intercepts for $g(x) = x^2 + 15x + 56$.

3 Relative Maxima and Minima

Definition 3.0.11 *A function f has a* ***relative maximum*** *(or* ***local maximum****) at c if $f(c) > f(x)$ when x is in an open interval around c (i.e. x is near c). It has a* ***relative minimum*** *at c if $f(c) < f(x)$ when x is near c.*

A function f has an ***absolute maximum*** *(or* ***global maximum****) at c if $f(c) > f(x)$ for all x in the domain of f. It has an* ***absolute minimum*** *at c if $f(c) < f(x)$ for all x in the domain of f.*

Things to notice: a hole in the graph cannot be a max or min. Neither can endpoints.

4 Sequences

Definition 4.0.12 *A sequence $\{a_n\}_{n=1}^{\infty}$ is* ***increasing*** *if $a_{n+1} > a_n$ for all n.*

Example 4.0.13 $\{1, 2, 3, \dots\}$ *is an increasing sequence.*

Definition 4.0.14 *A sequence $\{a_n\}_{n=1}^{\infty}$ is* ***decreasing*** *if $a_{n+1} < a_n$ for all n.*

Example 4.0.15 $\{-1, -2, -3, \dots\}$ *is decreasing.*

Definition 4.0.16 *A sequence $\{a_n\}_{n=1}^{\infty}$ is* ***bounded*** *if there exists a number M such that $|a_n| \leq M$ for all n. The number M is called a* ***bound*** *on the sequence.*

Example 4.0.17 $\{\frac{1}{n^2}\}_{n=1}^{\infty}$ *is bounded by 1. It is also bounded by anything bigger than 1.*
$\{1, 2, 3, \dots\}$ *is NOT bounded, since it keeps increasing towards infinity.*
$\{-1, -2, -3, \dots\}$ *is NOT bounded, since it keep decreasing towards $-\infty$.*

Practice Problems

Are the following sequences increasing, decreasing, or neither? Are they bounded? If so, find a bound.

1. $\{\frac{1}{n^3}\}_{n=1}^{\infty}$
2. $\{\frac{n^3}{n^2+2}\}_{n=1}^{\infty}$
3. $\{2n\}_{n=1}^{5}$